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Discrete photodetection and Susskind-Glogower ladder operators

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Abstract

We assert that state reduction processes in different types of photodetection experiments are described by using different kinds of ladder operators. A special model of discrete photodetection is developed by the use of superoperators which are based on the Susskind-Glogower raising and lowering operators.

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In order to give a quantum description of optical phase, Susskind and Glogower (SG) [1] have defined in the harmonic oscillator Hilbert space the following raising and lowering operators:

$$\hat{E}_+ = \sum_{n=0}^{\infty} |n+1\rangle\langle n|, \qquad \hat{E}_- = \sum_{n=0}^{\infty} |n\rangle\langle n+1|, \qquad \hat{E}_-|0\rangle = 0. \tag{1}$$

Here $|n\rangle$ ($n = 0, 1, \ldots, \infty$) are the number states. However, since the number spectrum is restricted from below by the vacuum state, the SG operators $\hat{E}_+$ and $\hat{E}_-$ are not unitary [1,2]:

$$\hat{E}_-\hat{E}_+ = \hat{1}, \qquad \hat{E}_+\hat{E}_- = \hat{1} - |0\rangle\langle 0|. \tag{2}$$

This nonunitarity leads to difficulties in the quantum description of optical phase [1,2]. This problem can be solved either by using a finite-dimensional Hilbert space [3] or by using the antinormal ordering of the SG operators $\hat{E}_+$ and $\hat{E}_-$ in the usual infinite-dimensional Hilbert space [4,5]. In the present work we would like to discuss another aspect concerning the SG operators $\hat{E}_+$ and $\hat{E}_-$. We will show that these operators can describe a state reduction process in a sort of discrete photodetection of the single-mode radiation fields represented by the density operators diagonal in the number state representation.

Usually, photodetection of the single-mode radiation field is described by the use of the mode annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$, which can be written in terms of the SG operators and the number operator $\hat{n} = \sum_{n=0}^{\infty} n|n\rangle\langle n|$ in the following form

$$\hat{a} = \sqrt{\hat{n}+1}\hat{E}_-, \qquad \hat{a}^\dagger = \hat{E}_+\sqrt{\hat{n}+1}. \tag{3}$$

When $\hat{a}$ and $\hat{a}^\dagger$ lower or raise a number state $|n\rangle$, they also generate the weight factor $\sqrt{n}$ or $\sqrt{n+1}$, respectively. The SG operators $\hat{E}_+$ and $\hat{E}_-$ only raise or lower the number states without generating any weight factor. This essential difference between the two types of ladder operators implies differences between photodetection schemes, in whose descriptions different types of ladder operators are used.

A usual model of continuous photodetection is the so-called closed-system model [6-9], in which both the radiation field and the photodetector are enclosed in a cavity, and the measurement is continuous. The density operator of the field is continuously reduced by the information provided by the photodetector. The instantaneous process of one-photon counting is described by the superoperator $\mathcal{J}$:

$$\hat{\rho}(t^+) = \mathcal{J}\hat{\rho}(t) \equiv \frac{\hat{a}\hat{\rho}(t)\hat{a}^\dagger}{\mathrm{Tr}\,[\hat{\rho}(t)\hat{a}^\dagger\hat{a}]}. \tag{4}$$

Here $\hat{\rho}(t)$ and $\hat{\rho}(t^+)$ are the density operators for the radiation field immediately before and after the detection. The superoperator $\mathcal{J}$ consists of nonunitary transformation (describing state reduction) and the normalization. The no-count process which occurs for a duration time τ is described by the superoperator $\mathcal{S}_\tau$:

$$\hat{\rho}(t+\tau) = \mathcal{S}_\tau \hat{\rho}(t) \equiv \frac{\exp(-\frac{1}{2}\lambda \hat{a}^\dagger \hat{a}\tau)\hat{\rho}(t)\exp(-\frac{1}{2}\lambda \hat{a}^\dagger \hat{a}\tau)}{\mathrm{Tr}\,[\hat{\rho}(t)\exp(-\frac{1}{2}\lambda \hat{a}^\dagger \hat{a}\tau)]}. \tag{5}$$

Here λ is a parameter characteristic of the coupling between the detector and the field. For a measurement, where an n-photon state is converted to an $(n-1)$-photon state whenever a photon is detected, the photodetection probability is proportional to n. This proportionality is described by the use of $\hat{a}$ and $\hat{a}^\dagger$ in equations (4) and (5) [9].

We suggest another photodetection scheme in which the single-mode radiation field is enclosed in a cavity. We send two-level Rydberg atoms in the lower state through the cavity, one after another, and measure their states at the exit. This experimental scheme is similar, from the technical point of view, to a micromaser [10]. However, we propose to use this system for realizing a special kind of photodetection. We use only radiation fields whose density operators are diagonal in the number state representation,

$$\hat{\rho} = \sum_{n=0}^{\infty} p(n)|n\rangle\langle n|. \tag{6}$$

When the measurement shows that one atom is excited, it means that one photon is subtracted from the radiation field. The detection of an excited atom is the only referred process in this model. Our idea is that in this photodetection scheme the field reduction is described by the superoperator $\mathcal{B}_-$ which includes the SG operators:

$$\hat{\rho}_{-1} = \mathcal{B}_- \hat{\rho} \equiv \frac{\hat{E}_- \hat{\rho} \hat{E}_+}{1 - \langle 0|\hat{\rho}|0\rangle} \tag{7}$$

where $\hat{\rho}$ and $\hat{\rho}_{-1}$ are the density operators for the radiation field before and after the subtraction of a photon. The normalization factor is $\mathrm{Tr}\,(\hat{\rho}\hat{E}_+\hat{E}_-) = 1 - \langle 0|\hat{\rho}|0\rangle$. In order to understand why equation (7) is valid we must show the differences between our model of discrete photodetection and the closed-system model of continuous photodetection. In discrete photodetection the measurement occurs only when an atom leaves the cavity, so that the number of measurements is equal to the number of atoms transmitted through the cavity. At that, the only referred measurement is that in which an excited atom is detected. Therefore in our model there is no analog to the no-count process of continuous photodetection. In continuous photodetection the measurement occurs at any time whenever the photodetector is active in the cavity, and the one-photon counting is referred as well as the no-count process. There the measurement is made inside the cavity by the interaction of the field with the detector. In our model the interaction between the field and the atoms is inside the cavity but the detector measures the states of the atoms outside the cavity, i.e., the detection is separated from the interaction with the field. Although the interaction in the cavity depends on the number of photons, the information obtained by us (the excitation of an atom) is independent of the features of interactions inside the cavity. The idea is that we are not interested in the properties of interactions inside the cavity and in the associated probabilities, it is of no concern to us how many atoms in the lower state we must send to obtain one of them in the excited state at the exit. By getting only the information that one

atom is excited we reduce an n-photon state of the radiation into an $(n-1)$-photon state in a way that is independent of n. This independence is described by the use of $\hat{E}_+$ and $\hat{E}_-$ in equation (7).

When the field is in the number state $|n\rangle$, the field reductions according to equations (4) and (7) are equivalent. Our model cannot be applied to the field states given by quantum mixtures of number states, $\sum_{n=0}^{\infty} C_n|n\rangle$. By getting only the information that one photon is absorbed we cannot conclude how the amplitudes C_n are changed. However, our model can be used for statistical mixtures of the form (6). For a state described by the density operator of the form (6), we have statistical probability $p(n)$ that the state is $|n\rangle$ but in fact only one of the states $|n\rangle$ exists in the cavity. As the result of state reduction (7), the changes in the photon-number distribution of the radiation field can be expressed in our model in the following form

$$p_{-1}(n) = \langle n|\hat{\rho}_{-1}|n\rangle = \frac{\langle n|\hat{E}_-\hat{\rho}\hat{E}_+|n\rangle}{1-\langle 0|\hat{\rho}|0\rangle} = \frac{p(n+1)}{1-p(0)}. \tag{8}$$

For comparison, the continuous photodetection model gives for the one-count process

$$p(n,t^+) = \langle n|\hat{\rho}(t^+)|n\rangle = \frac{\langle n|\hat{a}\hat{\rho}(t)\hat{a}^\dagger|n\rangle}{\langle \hat{n}\rangle_t} = \frac{n+1}{\langle \hat{n}\rangle_t}p(n+1,t). \tag{9}$$

The use of Bayes theorem [11,9] enables to obtain these results in a way that clarifies the principal differences between the two models. Bayes theorem can be written in the form

$$P(B_j|A) = \frac{P(B_j)P(A|B_j)}{\sum_j P(B_j)P(A|B_j)}, \tag{10}$$

where $P(B|A)$ is the conditional probability that event B occurs under the condition that event A is known to have occurred, and the mutually exclusive events B_j span the whole sample space: $\sum_j P(B_j) = 1$. Let event A be the detection of a photon and B_j be the fact that there is a certain number of photons in the cavity. In the continuous photodetection the probability that one of n photons in the cavity is detected during the time dt is $n\lambda dt$ and that no photon is detected is $(1-n\lambda dt)$. Therefore Bayes theorem (10) can be written, after it is known that one photon is detected at time t, as [9]

$$p(n,t^+) = \lim_{dt\to 0}\frac{p(n+1,t)(n+1)\lambda dt}{\sum_{n=1}^{\infty} p(n,t)n\lambda dt} = \frac{n+1}{\langle \hat{n}\rangle_t}p(n+1,t). \tag{11}$$

This equation is identical to equation (9). In our model the photodetection process occurs only when an atom is measured to be in the excited state, i.e., we refer only to the information that one photon is subtracted from the cavity. In this type of experiment, where we wait any time till we observe an excited atom, the probability $P(A)$ is equal to 1. Then Bayes theorem (10) can be written, after it is known that one photon was subtracted from the cavity, as

$$p_{-1}(n) = \frac{p(n+1)}{\sum_{n=1}^{\infty} p(n)} = \frac{p(n+1)}{1-p(0)}. \tag{12}$$

This equation is identical to equation (8).

The mean photon number immediately after the measurement of an excited atom can be easily calculated in our model by using equation (8) or (12) for the photon-number distribution. We get

$$\langle \hat{n} \rangle_{-1} = \sum_{n=0}^{\infty} n p_{-1}(n) = \frac{\langle \hat{n} \rangle}{1 - p(0)} - 1. \tag{13}$$

The denominator $1 - p(0)$ takes into account the fact that it is impossible to excite a photon from the vacuum. If the field was initially in the vacuum state, there is no photodetection process in our model and the number of photons in the cavity remains zero. With the exception of the vacuum-dependent factor, the mean photon number is merely reduced by 1 after the detection of an excited atom. The situation in the closed-system model of continuous photodetection is quite different. By using equation (9) or (11), one obtains the mean photon number immediately after the one-count process:

$$\langle \hat{n} \rangle_{t^+} = \sum_{n=0}^{\infty} n p(n, t^+) = \langle \hat{n} \rangle_t - 1 + \frac{(\Delta n)_t^2}{\langle \hat{n} \rangle_t}. \tag{14}$$

Here the photon-number variance is defined by $(\Delta n)^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$. This result shows that the the mean photon number of the post-measurement state depends on the pre-measurement photon statistics [8]. The difference between the mean photon numbers before and after the one-count process is not exactly equal to 1, but it has an additional term depending on the photon-number variance before the measurement.

We can generalize our model by sending atoms in the lower state through the cavity till the measurement shows a desired number N of excited atoms. It means that N photons were subtracted from the cavity. Then the field state is reduced according to

$$\hat{\rho}_{-N} = \mathcal{B}_-^N \hat{\rho} \equiv \frac{\hat{E}_-^N \hat{\rho} \hat{E}_+^N}{\mathrm{Tr}\,(\hat{\rho} \hat{E}_+^N \hat{E}_-^N)}. \tag{15}$$

Our experimental scheme also enables us to add photons to the cavity (this process is inverse to photodetection). In this case we send atoms in the upper state through the cavity, one after another, and measure their states at the exit till the measurement shows a desired number N of de-excited atoms. It means that N photons were added to the cavity. Then the field state is reduced according to

$$\hat{\rho}_{+N} = \mathcal{B}_+^N \hat{\rho} \equiv \hat{E}_+^N \hat{\rho} \hat{E}_-^N. \tag{16}$$

It is interesting to note that the transformation (16) is unitary and there is no need for a normalization factor. We find here a very special case where state reduction is described by a unitary transformation. The transformation (15) will be also unitary for the density operator $\hat{\rho}$ obeying the following condition

$$p(n) = \langle n | \hat{\rho} | n \rangle = 0 \quad \text{for} \quad n < N. \tag{17}$$

The mechanism described by equation (16) and by equation (15) under the condition (17) is number shifting, and we can refer to $\mathcal{B}_+^N$ and $\mathcal{B}_-^N$ as number-shifter superoperators. Then we have an analogy between the number shifter described by transformations (16) and (15) and the well known phase shifter described by the unitary transformation

$$\hat{\rho}_\phi = e^{i\phi\hat{n}}\hat{\rho}e^{-i\phi\hat{n}} \tag{18}$$

where phase shift ϕ is a real parameter.

In any real experiment we cannot ignore losses inside the cavity, and the detector of the atoms is never perfect. These experimental limitations introduce statistical features and thus destroy the state reduction mechanism which is based on the exact information. For imperfect detection we can generalize our model by assuming that the measurement reduces the density operator into the form

$$\hat{\rho}_{\pm\bar{N}} = \sum_N \alpha_N \mathcal{B}_\pm^N \hat{\rho}. \tag{19}$$

The detector efficiency distribution α_N must be sufficiently narrow around the true number $\bar{N}$ of excited (or de-excited) atoms in order to realize our model.

In conclusion, in the present work we have developed a special kind of discrete photodetection which is applicable to the single-mode radiation fields represented by the density operators diagonal in the number state representation. In this photodetection model state reduction is described by superoperators which are based on the SG raising and lowering operators.

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